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THE YUKON ECONOMY

ITS POTENTIAL FOR GROWTH AND CONTINUITY

VOLUME III A MODEL SIMULATION OF THE YUKON ECONOMY

by

D. W. Carr and B. M. Broadbent

D. Wm. Carr & Associates Ltd.

Canada

Background study prepared by
D. Wm. Carr & Associates Ltd.
as part of the
YUKON ECONOMIC STUDIES
undertaken for the
Department of Indian Affairs
and Northern Development
and the
Government of Yukon Territory

Ottawa

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PREFACE

The terms of reference for the comprehensive study of the economy of the Yukon Territory included, as one area for research, the "development of a mathematical model of the Yukon economy appropriate for the use of computer simulation techniques. Such a model would be used to test hypotheses developed under other sections of the study and to examine the consequences of various alternative federal and territorial policies and programs of action. It would also be used to indicate the impact of various investment programs and other stimuli which might arise from the private sector. It is anticipated that the model would be developed with sufficient thoroughness to remain a useful analytical tool for a number of years after the completion of the study." This is a report of the research carried out in appraising and specifying the model simulation of the Yukon economy. It is being carried out for the Department of Indian Affairs and Northern Development as part of an overall study of the potential economy.[1/]

1/ See the report, The Yukon Economy, Its Potential for Growth and Continuity by D. Wm. Carr and Associates Ltd., Ottawa, September 1968, henceforth called The Yukon Economy, Its Potential for Growth and Continuity.

In our Proposal for An Economic Study of the Yukon Territory prepared for the Department of Indian Affairs and Northern Development and the Government of Yukon Territory it was stated that "Because of deficiencies of data the study will be designed to progress by stages toward the construction of the desired model. Initially the study would concentrate on developing a simple but effective growth model for carrying out the above functions including those outlined in the terms of reference. It would be an aggregative model with a limited number of sectors, which could provide useful results even though the more detailed interindustry data was relatively inadequate. As a second stage in the study, the data available for preparing an interindustry input-output structure of the economy would be appraised, the data gaps analyzed and the ground work laid for compilation of the complete interindustry relations data necessary for a comprehensive input-output model of the Leontief type. The third step in the model construction would be to complete preparation of the interindustry relations data so that a full input-output model could be developed and brought into use. The evidence thus far would indicate that the inadequacies of the available data are such that this third step could not be initiated in the time span

of this study. Yet it may be found possible and it will be carried out if data and time permit. While it is desirable to leave the completion of an input-output model open at this time there is no doubt that research toward that goal can be advanced and delineated sufficiently so that future progress in construction of an effective input-output model can be rapid."

There is a major difficulty in specifying the form and structure of a simulation model for an economy for which the nature and extent of available data are little known at the beginning. This was the difficulty in the case of the Yukon model. Data were very limited when the studies began and until much of the research was completed, it was not clearly known how much additional data could be compiled. This was a serious handicap to the specification of the model at the beginning. But when it was found that much of the data necessary for the growth model simply could not be compiled, the range of choice in the models was clearly curtailed for the time being.

Early in the studies it was recognized that the simple aggregative growth model could not be effectively used for the functions required by the terms of reference for several reasons. A first reason was that much of the

data necessary to make such a model effective was not available (especially time series data) and such data as was available was not suitable for an aggregative model. As the general research progressed, it also became evident that the aggregative model was not the most suitable for analyzing the "consequences of various alternative federal and territorial policies and programs of action" because it would not show such effects on the key sectors of the economy and, at this time, an understanding of their impact on particular sectors of the economy would be of critical importance.

At the same time it was evident that much of the detailed information necessary for the suggested input-output model could not be adequately made available for some time and this disaggregative model could not be fully developed for the time being for the purposes required.

While the information on the Yukon economy was being compiled, much of it from primary sources, several steps were taken in exploring the structure and form of possible models and specifying the nature of the probable computer programs for the broad family of models that might be comprehended.

In this context, a paper was prepared by Professor May on November 5, 1967, entitled "Specification for a Program to Carry Out Simulation Exercises for the Proposed Yukon Model." This paper had the objective of

providing "sufficient information for a start to be made on designing the part of the computer program which will organize the outputting of the required information, even if the form and method of solution of the model itself are not yet determined. The accompanying macro flow chart showed in brief outline a particular arrangement of the processing required to provide the desired output." (p. 6)

Later, on February 17, 1968, a report was prepared by Professor May entitled "Outline of a Possible Model of a Region Characterized by Natural Barriers to Surface Transportation." This model was divided into two parts, an activity model and a transportation model. Its specification was oriented toward examining the problems of the spatial distribution of economic activities. Unfortunately, it required a great deal more data than it was possible to provide and since the objective was a working model as soon as possible, the deficiency of data was a serious handicap. In the course of the discussions on this model, many of the studies of statistical data were drawing to a close. It had become more clearly evident by this time that much of the required information would not be available for the time being. In these circumstances, it became evident that a much less complex model, than the activity-transportation formulation specified, would be more appropriate to the particular needs for the Yukon economy.

A second effort by Professor May was incorporated in the paper entitled "Specification of An Activity Model for the Yukon" dated July 3, 1968. This model had the disadvantage that it did not fully permit the range of applications desired in the terms of reference.

Professor May was only free up to July for the model study. In discussions with Dr. Wise following discussions with Messrs. MacArthur and Netherton it was concluded that the nature of the Yukon economy and the objectives of the model study warranted giving more consideration to an input-output model from the beginning. Accordingly, in this Report both the aggregative and disaggregative models have been appraised and conclusions drawn as to their relative effectiveness for the purposes required.

Ottawa,
October, 1968.

D. W. Carr

A MODEL SIMULATION OF THE YUKON ECONOMY

1. INTRODUCTION

This study has the objective of developing "a mathematical model of the Yukon economy appropriate for the use of computer simulation techniques."[1/] It is being carried out for the Department of Indian Affairs and Northern Development as part of an overall study of the potential of the Yukon economy.

The econometric model to be designed should have the capacity for use (a) in simulating the Yukon economy; (b) in testing hypotheses developed relative to the growth potential of the economy; (c) in examining the consequences of various alternative federal and territorial policies and programs of action; (d) in indicating the impact of various investment programs and stimuli which might arise from the private sector; and (e) in future years as an analytical tool.

1/ See Terms of Reference in Preface.

A model is, by definition, a representation or image of reality that is simplified according to the particular criteria used to indicate what is significant and what is not significant. The construction of a model of an economy may be expected to reflect these particular elements which are judged to be significant, the relations and measures presumed to be relevant to the problems of that economy and even the particular versions of economic theory that are judged to be pertinent to those problems.[1/]

It is generally recognized that in building econometric models there should be close integration of economic theory with the structural design of the model and with the related activities of data collection and processing. If this is most effectively done, then every econometric model is a special purpose model, designed for the particular and limited uses required. In other words, a model that is most appropriate for the Yukon will of necessity be considerably different from models constructed for most of the provinces in Canada, from models used by federal government agencies and from those used in private corporations. This is not just a difference due to fitting different parameters into the same logical structure. The logical structures themselves will differ.[2/]

1/ See also, "Some Remarks on an Econometric Model of a Provincial Economy" by T. I. Matuszewski, The Canadian Journal of Economics and Political Science, Vol. XXXI, No. 4, November, 1965, p. 552 et seq.

2/ Ibid.

In addition to this need to adapt the model to the significant economic theory, there is a second consideration relative to the application of such theory by way of an econometric model. If the most effective observation of the simulation is to be conducted, there must not be too much aggregation of the data for the model.[1/] In other words, it is desirable to work as close as possible to the micro-economic level in the model, since so much of the effective application of economic theory has been made in the field of micro-economics. To provide the soundest empirical basis for the economic analysis it is desirable that the data be as disaggregated as possible. Modern computation facilities make the handling of these disaggregated

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1/ See Guy H. Orcutt, Harold W. Watts, and John B. Edwards in "Data Aggregation and Information Loss," The American Economic Review, Vol. 58, No. 4, September, 1968, p. 785. "We discovered that, if the real world is similar to any one of the variants which were tried, use of less aggregated data than the national accounts data presently relied on would make possible: (1) the virtual elimination of small sample biases, (2) enormous improvements in the precision of estimates of parameters in macroeconomic models, and (3) greatly improved possibilities of detecting misspecifications and of correctly choosing between alternative formulations.

data generally quite feasible.[1/]

At the same time, the differences in the basic features or in the logical structures of econometric models designed for these special purposes does not prevent them being used co-operatively in conjunction with other models. Wide divergencies in finished models are quite compatible with close co-operation with other model builders especially at the level of collecting and adapting the data, that is in similarly shaping the

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1/ See "Some Remarks on an Econometric Model of a Provincial Economy" by T. I. Matuszewski, The Canadian Journal of Economics and Political Science, Vol. XXXI, No. 4, November 1965, p. 554. "We have failed to take full advantage of the possibilities of constructing disaggregated models. As well as being much more useful, such models are easier, not more difficult, to handle. It is not the number of variables and of relations that constitutes the problem today. In trying to squeeze a complex reality into a small number of complicated analytical relations of obscure meaning and suspect validity, we are trying to solve the hard way a problem that no longer exists. Computers handle with ease large volumes of numerical relations whose analytical form need not even be specified. High-speed replication opens the way to the study of amazingly involved deterministic or even stochastic interdependencies. Rapid access to large memories makes it possible to base the functioning of a model directly on vast masses of detailed information. It has now become possible to modify the data in the light of additional information even if it is only fragmentary, to make special adjustments, to vary the level of detail in any chosen part of the model. "Open-end" models, so to say, can now be constructed and used."

building blocks for the models, and in utilizing the results from one model in another. Moreover, the disaggregated econometric models discussed above, are ordinarily quite compatible with, and in fact require, a very large measure of co-operation in data collection, in the exchange of empirical results and in the pooling of technical knowledge.[1]

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[1] This has certainly been the case with the development of the input-output models for the Atlantic Provinces and for Quebec following the major work carried out in this field by the Dominion Bureau of Statistics.

2. CONSIDERATIONS IN SELECTING A MODEL

There are several major considerations to be taken into account in selecting the most effective type of model to represent the Yukon economy. The most critical of these considerations involve: (a) the nature of the Yukon economy; (b) the nature of the data available; and (c) the nature and purpose of the econometric analysis.

Nature of the Yukon Economy

The Yukon economy, like other northern economies, tends to import many of its inputs and export many of its outputs. It tends to depend on a relatively few industries, with mining providing the major part of the industrial output and with forestry and tourism providing a much smaller share. This lack of interdependence among its industries may be the greatest weakness of the economy. Its dependence mainly on mining tends to create an impermanence and uncertainty in the economy that leads to lack of stability and continuity. In the past decade, public and private investment expenditures have contributed much to the growth and stability of the

economy. The labour force tends to be mobile and in the mining industry a turnover of the work force three times a year is not uncommon. The population is relatively small (estimated at 18,000 in 1968) and has a high rate of participation in the labour force (43 per cent relative to 35 per cent for all Canada). With mining the major industry, the depletion of mines and abandonment of mining settlements may have substantial impacts ordinarily 25 to 50 years after the mine begins operations.

In these circumstances, it is evident that there are significant structural weaknesses in the Yukon economy. These weaknesses are related not only to the rate of growth of the economy but also to its stability and continuity as well as the permanence of the population and labour force.

Nature of the Data

With an economy as small as that of the Yukon and with the demand for its economic data arising only in very recent years, it may be expected that the data tend to be not available or inadequate. Much of the statistical information on the Yukon economy has been lumped in with that for British Columbia or the Northwest Territories. A special compilation of statistical data prepared as a part of the overall study of the Yukon economy has provided

much data on the Yukon, much of it for the first time.[1/] These statistics may provide the initial foundation from which some of the basic information for the econometric model can be developed. But major additions would need to be compiled and existing data would need to be appropriately adapted.

The extent of available data will be examined more fully in a later section of this report. It needs only to be emphasized here that the limitations of data will be very great in the beginning and that these data limitations will be greater for the aggregative models which usually require extended time-series data than for the disaggregative models which can ordinarily be quite effective using (and adapting) the data and coefficients for a particular year.

The Nature and Purpose of the Econometric Analysis

An effective mathematical (or numerical) model of an economy is one which comprehends all of the important structural relationships in that economy which are significant for the analysis of the areas or problems to be studied. In this context, econometrics is defined as

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1/ See Analysis of Statistics and Statistical Needs of the Yukon Territory, by Joan Gherson, a background study prepared as part of the Yukon Economic Studies, Ottawa, July, 1968.

the numerical science of economics, with the suffix implying a measurement of relationship.[1/] To accomplish its purpose, econometrics must first attempt to measure all the significant structural relationships encompassing the complete economy or a part of it. The set of all such relations constitutes an econometric model of that field of study. With such a model, econometrics can then calculate estimates of the effects of changes in certain variables or causal forces on the complete system. By separating its variables into exogenous (those impinging on the system from outside, and not significantly influenced in return by the system), endogenous (currently interdependent) and lagged endogenous, permits the model to be used as an aid to dynamic, short-run forecasts for the economy or for the firm or industry. Such forecasts are conditional on the correctness of the exogenous variables, which are forecast independently and outside of the econometric model framework.[2/]

In such prediction exercises, the knowledge of the structure of the economy can play an additional important role. Such knowledge can be applied, through the model, to the search for changes in activities or rules

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1/ See T. M. Brown, "Some Recent Econometric Developments" Canadian Journal of Economics and Political Science, Vol. 25, No. 1, February, 1959, p. 23.

2/ Ibid., p. 25.

which will deflect the course of the economy in the direction of the desired position or outcome. In this way, econometrics can serve as an aid in devising economic policy. Combined with such techniques as linear programming, econometric models can be effectively used in selecting optimum policies for particular objectives.

In the case of the Yukon economy, the central purposes of the econometric model analyses are: (a) to examine methods of strengthening the economy; (b) to appraise measures to increase its rate of growth and its stability; (c) to assess, in the course of this, the impact of public expenditure programs on employment, prices, productivity, private investment, total output and such.

In summary, the choice of the model most suitable for the study of the Yukon economy should be related to the problems of that economy, the nature of the questions to be answered about those problems and the limitations of the data available.

3. ALTERNATIVE MODELS

While there are many types and variations of models that may be appropriate for this analysis of the Yukon economy, this appraisal will be first confined to two broad alternatives which may be distinguished as the aggregative and disaggregative models. For this appraisal, it may be adequate to designate the aggregative model as one similar to that used by T. M. Brown in appraising the growth potential of Canada's economy for the Royal Commission on Health Services[1/] or an expansion of it now being used in the Department of Finance. The disaggregative model may be designated as one similar to the input-output models developed for the Atlantic Provinces, Alberta and Quebec.[2/] There are numerous other model types and modifications and gradations of these types but the distinction here designated will serve well as a basis for defining the advantages and disadvantages to be encountered in these two broad groups of models.

1/ See, Canadian Economic Growth, Ch. 9 and Appendices A, B and C, for detailed descriptions.

2/ A similar model is now being developed in Ontario.

Aggregative Models

The appraisal of each type of model can be considered under three headings: the general description of the type and use of the model; its data requirements relative to their availability; and its suitability for answering the pertinent questions concerning the Yukon economy.

Description of the Model. The aggregative model with the longest history of use in Canada is the Department of Finance model in Ottawa. It is used primarily for forecasting the effects of proposed fiscal policies. In 1966, it was using 70 equations to describe the structural relations in the Canadian economy. (The equations and glossary of symbols for this 1966 annual model are shown in Appendix A.) It derives its coefficients from a least squares analysis of time-series data covering the period from 1927 to the most recent year available. Its calculations are on the basis of 1957 dollars. Overall, the Department's databank for the model comprises 10,000 series about half of which are from Dominion Bureau of Statistics data and the other half from the Bank of Canada. These time series comprehend data similar to that shown in Brown's model[1/] covering population, employment, labour

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1/ Canadian Economic Growth, Appendix A.

participation, capital stocks, new investments, exports, imports, other components of gross national expenditure; national consumption, investment and trade balance; earned and disposable incomes; growth rates and other data. The <u>exogenous</u> data, that is, the data not generated within the model includes mainly such items as government spending, tax rates, monetary policy, exports, components of investment and the labour force.

<u>Data Requirements</u>. It may be evident that the data required for this aggregative model are quite extensive and must cover a substantial time period. Data of this kind and series is not available for the Yukon and it is unlikely that suitable data of this kind could be developed to cover a satisfactory time period. This model does not require information on industries or other disaggregative entities because it is not designed to examine these. For example, imports are treated only in terms of total import demand while, for the Yukon, imports should be divided into imports by industries and further broken down into services, capital goods, consumer goods, industrial materials and such.

<u>Suitability of the Aggregative Model</u>. The aggregative models described here are probably more suitable and efficient for a well-developed economy for which the extensive time-series data can be compiled. It is particularly

suitable for broad appraisals of the impact of public expenditures on aggregates such as income, savings, and consumption demand. But it does not show their impact on industries or other disaggregated parts of the economy.

For the Yukon, where the need is to examine ways of strengthening the economy, to improve its growth rate, its stability and its continuity, it would be desirable to examine the effects on the particular sectors of the economy in greater detail than this aggregative model would permit.

Disaggregative Models

Description. The model that will be used to describe the disaggregative technique will be the input-output model similar to those developed for the Atlantic Provinces, Quebec, Alberta[1/] and now being constructed for Ontario. There is no need to present here an extensive description of an appropriate input-output model for the Yukon since this is to be set out in a later section. The basic form of an input-output model is the input-output table. This table consists of a systematic accounting of transactions between sectors classified into industries, primary inputs and final demands for goods and services. These transactions are arranged in a tableau, or matrix, so that by

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1/ See The PARM System, An Appraisal, by D. W. Carr and S. J. May, Ottawa, 1967, a report prepared for Canada Emergency Measures Organization, p. 45 et seq. for a description of the work underway in Canada on such models in 1967.

reading along each row, the flow of output from a sector is distributed among its users which comprise other industries, government, household consumption, exports, etc. By reading down each column, the amount of each sector's inputs are distributed over the various sectors from which they originate, comprising other industries and primary inputs - the latter including households (labour), depreciation (capital consumption) and imports. Thus, each sector appears twice, once as a source of output, and again as a consumer of input.

It is desirable here to point out some significant features of the Yukon economy that would affect the structure of an input-output model. Because the Yukon is not a very diversified economy and is heavily dependent on imported inputs, the interdependence between sectors found in most economies is largely supplanted in the Yukon economy, except for services, by dependence on imports. Yet a greater understanding of the nature and implications of this dependence on imports is as important for the Yukon economy as the understanding of the interdependence between sectors is in a more diversified economy. The lack of interdependence within the Yukon and the great interdependence between the Yukon and outside sources of supply is a matter of critical significance in the analysis of the Yukon economy. It is also in this sphere that some pioneer work in input-output analysis can be contributed.

In addition, at this early stage in the development of the Yukon economy and in the development of the data describing its structural relationships, it may be presumed that it will contribute more to the understanding of the economy to see the effects of investment and other programs on the various sectors of the economy than on consumption, saving or similar variables such as may be shown by the aggregative model.

Data Requirements. The data for the industry and commodity input-output table for the particular year selected for the Yukon economy would be required to show: (a) the distribution of imported goods and services by industries in the year selected; (b) the industry of origin of domestically-produced materials, supplies and services which were used by the various industries; (c) the incomes earned by the factors of production in each industry; and (d) the industry of origin of final output.[1/] Conceivably, the table could be set up in terms of physical units of commodities and services, but because of the desirability of having input and output for an industry add to the same total, the items are expressed in terms of prices for a given year.

1/ See Supplement to the Inter-Industry Flow of Goods and Services, Canada, 1949, Cat. No. 13-513, Dominion Bureau of Statistics, Ottawa, 1960, p. 11.

When the input-output transactions table has been constructed for a given year, as set out in the Description above, a table of input or technical coefficients can be developed from it. By a technical coefficient is meant the amount of inputs required from each industry or inputs required to produce one dollar's worth of the output of a given industry. These also represent the direct purchases that will be made by a given industry from other industries or imports for each dollar's worth of current output. But when there are additional sales of the output of an industry to the final demand sector, there will be additional or indirect increases in the output of all industries and in imports and these when added to the direct coefficients provide the bases for a third table, the inverse of the structural matrix.

Once a general solution or table of direct and indirect coefficients has been obtained, the input-output model can be used for a variety of analytical purposes. These uses will be examined in a later section.

This broad description outlines the nature of the data required. The economic study of the Yukon[1/] has provided an initial foundation from which to proceed in obtaining the necessary data. But much more work would

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1/ See The Yukon Economy, Its Potential for Growth and Continuity.

be required to provide the additional detailed data on the structure of industries and other parts of the Yukon economy. These data requirements are examined in detail in a later section. A great contribution of the research required in compiling this data is that in the course of it there will also be gathered the information and insights into the detailed structure of the economy necessary to understand its particular relationships and the effects of various public policies on its structure, stability and growth in output.

Our review of the data likely to be available for this purpose would suggest that data for the year 1966 would be most comprehensively available, timely and effective for such an input-output table.

Suitability of the Disaggregative Model. The following points are significant to the suitability of the input-output model:

(1) The input-output table of transactions provides an excellent description of the demand and supply relationships of an economy in equilibrium. In addition to the description of these structural relationships valuable insights into the unique features of an economy can be provided by the data research. Such an input-output table describes the economy as

it is, not as it should or might be. On this basis, the model can achieve comprehensiveness with a minimum of arbitrary decisions or mechanistic procedures.

(2) If input-output tables are available for two or more regions it is possible to make a very useful comparative analysis. For example, a comparison, by means of input-output tables, of the Yukon economy with those of Alberta, the Atlantic Provinces, Quebec and Ontario should not only demonstrate some unique features of the Yukon economy but is likely to suggest several avenues to test techniques and policies by which it may be strengthened. Such an input-output comparison is particularly valuable for a relatively underdeveloped economy like that of the Yukon. It could indicate sectors of greatest growth potential and sectors to be strengthened if such optimum growth is to be attained.

(3) Input-output analysis provides a comprehensive and effective tool for projection and forecasting. It may be compared with two other model approaches to forecasting - partial and by simultaneous equations.

The partial forecast usually involves the projection of one or more time series and in its simplest form fits a mathematical curve to an individual time series and extrapolates this curve to some future date.

This is a rudimentary technique which works well for only a few relatively stable time series, may be quite ineffective in the short run and often gives rise to inconsistencies when several time series are separately projected.

Projection by simultaneous equations similar to the Department of Finance model (see Appendix A) removes the risk of inconsistencies but since they are based on only a limited number of time series they still provide only a partial forecasting. Extensive aggregation of the time series can improve consistency and provide comprehensiveness but the aggregation limits their usefulness and prevents examination of the micro-impact on particular industries or sectors.

Projections using an input-output transactions table are usually termed "consistent forecasting" because the output of each industry is consistent with the demands, both final and from other industries, for its products. It is also possible to refine the forecast by adapting the production coefficients to productivity changes resulting from the new capital investments. Thus the input-output transactions forecasts have several major advantages over the partial or simultaneous equations methods.

(4) Input-output analysis permits a detailed and extensive examination of the impact or multiplier effect of a given addition to capital investment or incomes. These multipliers are related to the marginal propensity to consume. By input-output analysis it is possible to derive sectoral multipliers which will indicate the sectors generating the greatest multipliers. Aggregate multipliers can also be calculated. Both direct and indirect multiplier effects can be examined. In similar ways, employment multipliers can be computed by means of the input-output table.

(5) Input-output analysis can be useful for public policy-makers and the business community by its contributions to sensitivity analysis and feasibility tests. Consistent forecasts, such as those input-output analysis can provide, are basic to sensitivity analysis. The objective of sensitivity analysis is to determine those components of the economy which are most sensitive to alternative patterns of growth stimulated by alternative public spending policies. Sensitivity studies also deal with the analysis of the effects of change in the import-export patterns on employment and production.

Feasibility tests are closely allied to sensitivity studies. The feasibility of achieving a certain level of production or employment in the economy by a given target date might be examined, for example, in terms of required final demand and interindustry relationships. The feasibility of obtaining domestically the resources required to achieve the composition of products inherent in a particular projected level of final demand might be another example. For effective long-run planning, such feasibility tests need to be made and they need to be accompanied by suitable adjustments in the basic structure of the economy which are appropriate to a dynamic economy in an era of technological change, including changes in the relative productivity of particular industries over time.

(6) The input-output model can be used very effectively for regional and interregional input-output analysis. This is particularly important for consideration of the Yukon economy because with so large a part of the inputs arising from imports from the rest of Canada the impact of Yukon expansion on the rest of the economy can be very great. It is appropriate that this multiplier effect be measured if Canada continues to contribute so heavily to investment and growth in the Yukon economy.

(7) The simple static input-output model can be modified into a dynamic model by adding a table of incremental capital coefficients and a few other refinements.

The above seven features describe broadly the range of applications of the input-output model. It may be useful to set out some examples of specific applications which are likely to be most helpful relative to the Yukon economy.

(1) By the input-output analysis it is possible to trace the effects of changes in public investment or final demand on the gross level of output of each industrial sector and on incomes, employment, tax yields and such. These investigations can be made without resorting to the unsatisfactory techniques of projecting past trends or making intuitive judgements.

(2) Input-output analysis can provide information on the relationship between federal revenues arising from a particular economic activity in the region and the federal spending on transfer payments and on purchase of goods and services in the region.

(3) Input-output analysis permits comparison of the results, both direct and indirect, in employment and incomes generated by investment in alternative industries which vary in their labour and capital intensiveness.

(4) Current questions that might be answered by this model would include:

- How many jobs and how much income depends on the export of mineral products?
- How many jobs and how much income depend on federal expenditures in the Yukon? How much of this stays in the Yukon?

(5) Projective questions that might be answered would include:

- What would be the effects on the Yukon economy of establishment of a fully developed smelting industry?
- What might be the effect on the coefficients of production and on interindustry relationships of alternative measures to strengthen the economy - by improved services, by industrial investments?

Summary on Choice of Alternative Models

The weight of evidence indicates the disaggregative or input-output model would have many advantages over the aggregative model for the study of the Yukon economy. The opportunities for either detailed or comprehensive analysis are clearly much greater with the input-output approach. The necessary time-series data for the

aggregative model are less readily available than the data for a particular year for the disaggregative model. Such time series could not be made available for more than several years at best and this would be too small a sample to give dependable results in the aggregative model. Yet even if such time-series data were available for a sufficiently long period, the aggregative model could provide only aggregative information such as on total income, employment or such. It could not provide the insights into the structure of the Yukon economy and its weaknesses that are so necessary to improving its development potential. The input-output model could provide these structural insights and it could also provide much other useful information as well as aggregative information on incomes, employment and other data similar to that provided by the aggregative model.

4. NATURE AND SPECIFICATIONS OF THE INPUT-OUTPUT MODEL

Input-output tables may be developed or adapted to suit particular applications but the same basic principles apply to all of them. An input-output table consists of a systematic accounting of transactions between sectors that may be classified into industries, primary inputs and final demands for goods and services. The transactions are arranged in a tableau or matrix so it shows how the output of each industry is distributed among the other industries and sectors of the economy. At the same time it shows the inputs to each industry from other industries and sectors. A hypothetical input-output or transactions table is illustrated in Table 1.[1/] This table is highly simplified, containing only six hypothetical industries (against 120 industries and 400 commodities in the latest input-output table prepared by the Dominion Bureau of Statistics)[2/] but it is realistic in other respects.

1/ Adapted from William H. Miernyk, The Elements of Input-Output Analysis, New York, 1967, p. 9.

2/ See The PARM System, An Appraisal, p. 45.

TABLE 1

HYPOTHETICAL TRANSACTIONS TABLE

Industry Purchasing

Industry Producing	Outputs 1/ → / Inputs 2/ ↓	(1) A	(2) B	(3) C	(4) D	(5) E	(6) F	(7) Gross inventory accumulation (+)	(8) Exports to foreign countries	(9) Government purchases	(10) Gross private capital formation	(11) Households	(12) Total Gross Output
		Processing Sector						Final Demand					
Processing Sector	(1) Industry A	10	15	1	2	5	6	2	5	1	3	14	64
Processing Sector	(2) Industry B	5	4	7	1	3	8	1	6	3	4	17	59
Processing Sector	(3) Industry C	7	2	8	1	5	3	2	3	1	3	5	40
Processing Sector	(4) Industry D	11	1	2	8	6	4	0	0	1	2	4	39
Processing Sector	(5) Industry E	4	0	1	14	3	2	1	2	1	3	9	40
Processing Sector	(6) Industry F	2	6	7	6	2	6	2	4	2	1	8	46
Payments Sector	(7) Gross inventory depletion (-)	1	2	1	0	2	1	0	1	0	0	0	8
Payments Sector	(8) Imports	2	1	3	0	3	2	0	0	0	0	2	13
Payments Sector	(9) Payments to government	2	3	2	2	1	2	3	2	1	2	12	32
Payments Sector	(10) Depreciation allowances	1	2	1	0	1	0	0	0	0	0	0	5
Payments Sector	(11) Households	19	23	7	5	9	12	1	0	8	0	1	85
	(12) Total Gross Outlays	64	59	40	39	40	46	12	23	18	18	72	431

1/ Sales to industries and sectors along the top of the table from the industry listed in each row at the left of the table.

2/ Purchases from industries and sectors at the left of the table by the industry listed at the top of each column.

In the case of the Yukon, the significant sectors are relatively few - mining, electric power, construction, retail trade, transportation and service trades comprise all but a small part of the output.

To put the transactions into comparable units they are valued in terms of producers' prices while the gross margins are attributed to the combined transportation, storage and trade industry in payment for time and place utility.

A great deal of work is involved in constructing an input-output table but once constructed it can be readily interpreted. Each row (reading from left to right, Table 1) shows the output sold by each industry or sector along the left-hand side of the table to each industry or sector across the top of the table. Each column (reading from top to bottom) shows the purchases made by each industry or sector along the top of the table from the industries and sectors along the left-hand side. Since this is a square table, there is one row to correspond to each column.

The relationships can be illustrated by an example. Consider the relationship between industry E (row 5 and column 5) and industry C (row 3 and column 3). To find the share of industry E's output sold to industry C, read

across row 5 until it intersects column 3. We see that industry E sold one billion dollars' worth of goods to industry C during the period covered by the table. To find how much industry E buys from industry C, go over to column 5 and read down until this column intersects row 3. We see that industry E bought from industry C products worth five billion dollars. Hence the net transaction between industries C and E during this period is four billion dollars in favour of industry C.

The Parts of the Table[1]

1. The Processing Sector. The upper left-hand corner of the table has been set off in heavy double lines and labeled the processing sector. This is the sector of an input-output table which contains the industries producing goods and services. Among them we would generally find agriculture, various manufacturing industries, transportation, communications and other utilities, wholesale and retail trade, the service industries, construction, and as many other industries as are isolated for separate treatment in the table. This is the portion of the hypothetical table that is highly simplified, and in practice we would expect to find this sector expanded to 50 or more industries, thus greatly expanding the size of the entire table.

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[1] This section has been mainly adapted from The Elements of Input-Output Analysis by William H. Miernyk.

2. The Payments Sector. On the left-hand side of the table, rows 7 to 11 are set off under the heading payments sector. This sector includes these five rows read all the way across the table. We shall examine each of the five parts of the payments sector in turn.

(a) Row 7, gross inventory depletion. By gross inventory depletion we mean the using up of previously accumulated stocks of raw materials, intermediate goods, or finished products. Thus in row 7, column 2, we see that during the period covered by the table industry B used up two billion dollars' worth of the stock it had put into inventory in an earlier period. The amount of inventory depletion in all other industries and sectors can be found by reading down each column until it intersects row 7.

(b) Row 8, imports. To find the value of imports purchased by each industry and sector, read down each column until it comes to row 8. This procedure shows, for example, that industry E imported three billion dollars' worth of goods from abroad, while industry D imported nothing.

(c) Row 9, payments to government. For simplicity, assume that payments to governments (federal, provincial and local) in the form of taxes, represent purchases of

government services such as police and fire protection, maintenance of the armed forces, and similar services which most of us take for granted. Although there is no direct correspondence between payments to government and the amount of government services provided to each industry (because, for example, how do you "value" the protection of the armed forces ?), it will simplify matters if we assume that the figures in row 9 represent the value of government services to each of the industries and other sectors listed across the top of the table.

(d) Row 10, <u>depreciation allowances</u>. Reading across row 10 we see the amounts of depreciation allowances set aside by each of the industries listed across the top of the table. These numbers approximate the cost of plant and equipment used up in the production of the goods represented in this table. Note, for example, that industry A (column 1) allowed one billion dollars during the period covered by the table for the depreciation of machinery and other equipment.[1/]

(e) Row 11, <u>households</u>. This row represents the wages, salaries, dividends, interest, and similar payments made to households by each of the industries and other sectors

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1/ An input-output table is compiled for a given time period. In practice this is usually a calendar year. There is no reason, however, why the period could not be either longer or shorter than a year.

listed across the top of the table. We have inserted fairly large figures in this row to indicate in particular the relative importance of payments to labour in our hypothetical economy. Industry A paid out 19 billion dollars in the form of wages, salaries, and other forms of household income; industry B paid out 23 billion dollars, and so on across row 11.

3. The Final Demand Sector. The final demand sector consists of columns 7 through 11 read all the way down the table. The final demand sector is of special importance because it is the autonomous sector - the one in which changes occur which are transmitted throughout the rest of the table. It is here that the transactions which will be discussed presently originate. We will describe each of the parts of this sector briefly.

(a) Column 7, gross inventory accumulation. This column shows the amounts of additions to inventories held by each of the industries and sectors along the left-hand side of the table. During any given time period some of the goods produced do not get into the hands of their final consumers. Retailers must stand ready to provide consumers with a variety of goods at all times. Hence they must keep a stock of goods on their shelves. Wholesalers must likewise be ready to ship to retailers upon short notice. And manufacturers

will usually have a stock of the goods they produce on hand at any given time. Column 7 shows the amounts of inventories accumulated during the period covered by the table regardless of where those inventories are held, whether at the factory, in warehouses, or in retail establishments.

(b) Column 8, exports. This column shows the value of exports from each of the processing industries and other sectors during the period covered by the table. Note that industry A in our hypothetical economy exported five billion dollars' worth of goods while households exported nothing. This would be typical of a national table since residents of one country ordinarily do not sell their labour services in another country. In regional applications, however, households can export labour services across regional boundaries, and it is also fairly common for management and technical consulting services to be exported from one region to another.

(c) Column 9, government purchases. Purchases made by levels of government are given in this column. The entry where the government column and the government row intersect indicates that there are some intragovernmental transactions, just as there are transactions within other industries and sectors included in our table.

(d) Column 10, gross private capital formation. This column shows the amount of sales from each industry or sector along the left side of the table to buyers who use their purchases for private capital formation. All entries in the transaction table, except those in column 10, are on current account. Purchases by all buyers for the replacement of or additions to plant and equipment - and any other purchases which are entered on capital account - are summarized by the entries in column 10. Viewed another way, each entry in column 10 can be considered an input from the industry or sector listed at the left to the Gross Private Capital Formation "industry."

(e) Column 11, households. The entries in this column represent purchases of finished goods and services by their ultimate consumers from the industries and other sectors along the left hand side of the table.

4. Total Gross Output and Total Gross Outlay. The final row and the final column of the table have yet to be explained.

Row 12, total gross outlay, shows the total value of inputs to each of the industries and sectors in each column at the top of the table. The total value of purchases by industry A, for example, is 64 billion dollars the amount of the entry in row 12, column 1.

The input-output table is essentially a system of double-entry bookkeeping. Within each industry in the processing sector all of the receipts from sales are paid out for goods and services purchased from other industries or sectors. It might help to think of these as payments to factors of production. Some of the receipts are paid to the government in taxes, and some might be added to capital account. But the receipts from all outputs will just balance total outlays for each industry. After taking into account appropriate inventory changes, the total gross output, column 12, of each industry in the processing sector is equal to the total outlays made by that industry. Thus in the hypothetical table, the first six entries in the Total Gross Output Column are identical with the first six entries in the Total Gross Outlay row.

This is not true of the totals in the remaining rows and columns, however. We would not expect imports and exports to be exactly equal in any given year. Nor are inventory depletions and inventory accumulations likely to be the same during a given time period. Similarly, one would not expect a balance between government purchases and payments to governments, capital spending and depreciation allowances, and payments to and by households in the same year. But the individual differences must "cancel out" when we view the entire economy. As is true of any

single processing industry, total outlays must equal total outputs for the economy as a whole. The total of all rows in the payments sector must equal the total of all columns in the final demand sector for the same reason that the Gross National Product computed from the product side must equal Gross National Product computed from factor payments.

One last point may be raised before tracing through a set of transactions. How does the Total Gross Output (or Total Gross Outlay) in the input-output table compare with Gross National Product? They are not the same. The GNP is defined as "the current market value of final goods and services produced in a given year." But even for the same year, GNP will not be the same as the Total Gross Output of an input-output table. In computing GNP every effort is made to eliminate double-counting. But since the input-output table measures all transactions in the economy the value of goods and services produced in a given year is counted more than one time; that is, we deliberately double-count.

The objective is different in the two cases. In national-income analysis the object is to measure the final value of goods and services produced by the entire economy in a given year. We obviously wish to count one time only each good and service produced. In the input-output table, however, we wish to account for all

transactions. Since some goods will enter into more than one transaction, their value must be counted each time a different transaction takes place. What we have then is an accumulation of value added at each stage of the production process until a good gets into the hands of its final consumer. Input-output analysis and national-income accounting are not two separate branches of economics, however.

There is nothing rigid about the classifications used in the payments and final demand sectors of the hypothetical transactions table. The industries in the processing sector can be disaggregated to any degree desired - within the limit of data availability. Similarly, the payments and final demand sectors can be split into more rows and columns than those shown in Table 1. For example, the import row (and export column) can be disaggregated along geographic lines. Instead of a single government row (and column) there can be three, one each for federal, provincial and local governments. And the household row (and column) could be further divided; for example, on the basis of income distribution. The input-output table is a flexible analytical tool. It can be made as detailed or as condensed as necessary for any any given purpose. The only limitation is that there

must be one row for each column in the processing sector. It is convenient, although not necessary, to have a final demand column for each row in the payments sector.

There is no fixed rule for including (or excluding) any specific economic activity in the final demand (or payments) sector. Table 1 illustrates a relatively "open" input-output model. For some purposes it might be desirable to "close" the system with respect to one or more of the activities in the final demand (payments) sector. Households, for example, can be shifted into the processing sector, and the same is true of any other activity in final demand. Similarly, some activities normally included in the processing sector can be shifted to final demand. The construction and maintenance industry can be included in final demand, for example, if one is interested in analyzing the interindustry effects of changes in construction activity. The decision of how "open" or "closed" an input-output table is to be depends largely upon the purpose for which it is to be used. Our hypothetical example illustrates a general-purpose, open, nondynamic input-output system. But it must be emphasized that the basic model can be altered in a number of ways, depending upon the analytical use for which it is intended.

Direct Purchases and Technical Coefficients

After an input-output table of transactions has been constructed for a given year, a table of input or technical coefficients can be developed from it. By a technical coefficient we mean the amount of inputs required from each industry to produce one dollar's worth of the output of a given industry. Technical coefficients are calculated for processing sector industries only, and may be expressed either in monetary or physical terms. Our hypothetical table is expressed in cents per dollar of direct purchases.

Two steps are involved in the calculation of technical coefficients: Gross output is adjusted by subtracting inventory depletion during the period covered by the table to obtain adjusted gross output. Since gross outlays in the processing sector are identical with gross outputs in this sector, adjusted gross outputs in our hypothetical economy can be computed by subtracting the entries in row 7 from the entries of row 12 of Table 1. The results can then be entered as a new row at the bottom of the table. The second step in the calculation of technical coefficients consists of dividing all the entries in each industry's column by the adjusted gross output for that industry.

For example, the transactions table for Canada, highly aggregated as to industries, may be illustrated by Table 2. Computing the coefficients for this transactions table provides us with Table 3. If these technical coefficients remain constant or if they can be adjusted on the basis of new information it is possible to calculate the amount of direct purchases required from each industry along the left-hand side of Table 3, as a result of an increase or decrease in final demand for the output of one or more industries.

Stability Conditions for the Table of Technical Coefficients

The table of direct coefficients by itself is of limited usefulness because it shows only the initial effects of a change in the output of one industry on the industries from which it purchases inputs. This table forms the basis for a general solution of an input-output problem to be examined in the next section. Because of this it is necessary that the table of direct coefficients meets certain stability conditions, namely, (a) that at least one column in the table adds to less than unity, and (b) that no column in table adds to more than unity. The mathematical proof of these conditions is quite complex and will not be

TABLE 2

CONDENSED INPUT-OUTPUT TABLE, CANADA, 1949

	Primary Industries	Manufacturing	Construction	Services	Final Demand	Total
	(millions of dollars)					
Primary Industries	89.4	1660.6	33.2	97.8	1989.0	3870.0
Manufacturing	549.6	2487.5	1027.4	1194.5	6714.0	11973.0
Construction	65.5	61.5	2.7	677.6	2166.0	2979.0
Services	382.4	1949.2	667.8	2004.9	7040.0	12042.0
Primary Inputs	2783.1	5814.2	1248.7	8067.2	2246.0	20156.0
	3870.0	11973.0	2979.0	12042.0	20156.0	

Source: *Inter-Industry Flow of Goods and Services, Canada, 1949*. Dominion Bureau of Statistics, Cat. No. 13-513, Ottawa.

TABLE 3

INPUT-OUTPUT COEFFICIENTS

(Dollars of inputs per dollar of output)

	Primary Industries	Manu-facturing	Con-struction	Services
Primary Industries	.023	.139	.011	.008
Manufacturing	.142	.208	.345	.100
Construction	.017	.004	.001	.056
Services	.099	.163	.224	.166
Primary Inputs	.719	.486	.419	.670
	1.000	1.000	1.000	1.000

Source: Calculated from Table 2.

demonstrated here.[1/] It is sufficient to note that, when the table is expressed in monetary values, it is evident that an industry cannot pay more for its inputs than it receives from the sale of its outputs. Moreover, the steps described above for computing input coefficients in the open, static model show that these conditions will be met if in each column the sum of entries in the payments rows (less the inventory row) is greater than inventory depletion. In practice, these entries are relatively large and the stability conditions are ordinarily safely met.

Direct and Indirect Coefficients

The direct coefficients do not represent the total addition to output resulting from additional sales to the final demand sector. An integral part of input-output analysis is the construction of a table which shows both direct and indirect effects of changes in final demand (see Table 4). There are various methods for computing this combined direct and indirect effect. The iterative or step-by-step method is used here for illustration only, and will not include all the calculations required to construct a table.

1/ For a proof in the case where all technical coefficients are positive see Robert Solow, "On the Structure of Linear Models," Econometrica, XX (January 1952), 29-46. See also Carl F. Christ, "A Review of Input-Output Analysis" in Input-Output Analysis: An Appraisal (Princeton: Princeton University Press, 1955), p. 148-49.

If it is assumed that there is a one-dollar increase in the demand for the products of primary industries (Table 3), this will increase intraindustry transactions by 2.3 cents (see row 1, column 1). Thus the gross output of primary industries will increase by at least \$1.023. But for the output of primary industries to increase in this way, the firms in these industries must increase their purchases from the other industries (manufacturing, construction, etc.). Sales from manufacturing to primary industries will increase by an additional 14.5 cents (\$1.023 x 0.142) as a result of the increased output in primary industries. Similarly, sales from construction will increase by 1.7 cents (\$1.023 x 0.017) and so on down column one of Table 3. Yet the indirect effects do not end here. When manufacturing expands its production to meet the increase in requirements of the primary industries, the increased demand thus generated will affect all other industries which provide inputs for manufacturing. In this way, calculations similar to those above could be repeated to include in a sequence the inter-industry effects of the increased demand and these changes could then be added to build a table showing the total requirements, direct and indirect, resulting from the delivery of one dollar's worth of the products of each industry to the final demand sector.

There is of course an alternative method of calculating this which uses high-speed computing equipment to provide the same results quickly. In technical terms this method involves taking the difference between an identity matrix and the input coefficient matrix (Table 3) and from this computing a transposed inverse matrix. This table (Table 4) shows the <u>total</u> requirements, direct and indirect, per dollar of delivery outside the industry sectors, that is, to final demand.

<u>Stability Condition</u>

There is a basic condition for stability that must be met by the table of direct and indirect requirements. This condition is that there can be no negative entries in the table of direct and indirect requirements. Consideration shows this to be a logical condition.

Each row of Table 4 shows the output directly and indirectly required from each sector at the top of the table to support the delivery of $1.00 to final demand by the sector at the left of each row. Each <u>column</u> shows the output required for a <u>single</u> sector (directly and indirectly) to support $1.00 of delivery to final demand by each of the industrial sectors.

TABLE 4

INVERSE OF STRUCTURAL MATRIX

(Inputs per dollar of final demand)

	Primary Industries	Manu-facturing	Con-struction	Services
Primary Industries	1.057	.194	.087	.039
Manufacturing	.224	1.345	.511	.198
Construction	.029	.025	1.026	.072
Services	.177	.293	.386	1.262
Primary Inputs	0.998	0.997	0.999	0.999

Source: Calculated from Table 3.

Table 4 is a general solution of the input-output system. It illustrates the principal of economic interdependence. In the Yukon this is most evident with the service industry. The table can be used to show how a change in demand for the output of one sector stimulates production in other sectors. It shows the end result after all of the "feedback effects" have worked themselves out. The model illustrated here is static rather than dynamic. It would be necessary to introduce time lags to achieve precisely the equilibrium results given in Table 4.

When the general solution or table of direct and indirect coefficients has been obtained, the input-output model can be used for a variety of analytical purposes, some of which were outlined in a previous section.

5. <u>THE INPUT-OUTPUT SYSTEM - A SYMBOLIC SUMMARY</u>

The static, open input-output system can be summarized in symbolic language. Basically, the input-output model is a general theory of production. All components of final demand are considered to be data. The problem is to determine the levels of production in each sector which are required to satisfy the given level of final demand.

The static, open model is based upon three fundamental assumptions. These are that:

1. Each group of commodities is supplied by a single production sector.
2. The inputs to each sector are a unique function of the level of output of that sector.
3. There are no external economies or diseconomies.

The economy consists of <u>n + 1</u> sectors. Of these, one sector - that representing final demand - is autonomous. The remaining <u>n</u> sectors are nonautonomous, and structural interrelationships can be established among them.[1/]

1/ Otherwise stated final demand, for each sector, is an <u>exogenous</u> variable, and the interindustry transactions are <u>endogenous</u> variables.

Total production in any one sector during the period selected for study may be represented by the symbol X_i. Some of this production will be used to satisfy the requirements of other nonautonomous sectors. The remainder will be consumed by the autonomous sector. This situation may be represented by the following balance equation:

$$(1) \quad X_i = X_{i1} + X_{i2} + \ldots + X_{in} + X_f \ (i = 1 \ldots n)$$

where X_f is the autonomous sector, and the remaining terms on the right-hand side of the equation are the nonautonomous sectors in the system.

Assumption (2) above states that the demand for part of the output of one nonautonomous sector X_i by another nonautonomous sector X_j is a unique function of the level of production in X_j. That is:

$$(2) \qquad X_{ij} = a_{ij}X_j$$

Substituting (2) in equation (1) yields

$$(3) \quad X_i = a_{i1}(X_1) + a_{i2}(X_2) + \ldots a_{in}(X_n) + X_f \ (i = 1 \ldots n)$$

This may be written more compactly as

$$(4) \qquad X_i = \sum_{j=1}^{n} a_{ij}(X_j) + X_f \ (i = 1 \ldots n)$$

where X_j is the amount demanded by the jth sector from the

$_i$th sector, and X_f represents the end-product (final) demand for the output of this sector. The model can be illustrated schematically in Figure 1.

From the transactions table (Table 2) the technical coefficients are computed (Table 3). These coefficients show the direct purchases by each sector from every other sector per dollar of output. They are given in equation (2) above, which may be rewritten as:

$$(5) \qquad a_{ij} = \frac{X_{ij}}{X_j}$$

The coefficients are computed for the processing sector only in two steps:

(1) Inventory depletion during the base period is subtracted from total gross output to obtain adjusted <u>gross</u> output.

(2) The entry in each column of the processing sector is divided by adjusted gross output to obtain the a_{ij} shown in (5). This gives the following matrix of technical coefficients.

$$(6) \qquad A = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ a_{n1} & & a_{nj} & & a_{nn} \end{bmatrix}$$

FIGURE 1

SCHEMATIC REPRESENTATION OF THE TRANSACTIONS TABLE OF A STATIC, OPEN INPUT-OUTPUT MODEL

Industry Purchasing → ↓ Industry Producing		X_f	X_i
	$\sum_{j=1}^{n} a_{ij} (X_j)$ Processing Sectors Processing Sectors	Final Demand	Total Gross Output
$X_p \equiv X_f$	Payments Sectors		
$X_o \equiv X_i$	Total Gross Outlays		

As noted in the preceding section, the table of direct and indirect requirements per dollar of final demand is obtained by inverting a Leontief matrix, which is defined as $(I - A)$. The new matrix of coefficients showing direct and indirect effects (Table 4) is generally transposed to obtain $(I - A)_T^{-1}$. This matrix may be designated as R.

$$(7) \qquad R = \begin{bmatrix} r_{11} \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \vdots & & \vdots \\ r_{i1} \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \vdots & & \vdots \\ r_{n1} \cdots & r_{nj} & \cdots & r_{nn} \end{bmatrix}$$

Analytically, the input-output problem is that of determining the interindustry transactions which are required to sustain a given level of final demand. After a transactions table has been constructed, we can compute the A and $(I - A)_T^{-1}$ matrices. For any new final demand sector inserted into the system, we use these to compute a new table of interindustry transactions as follows:

$$(8) \qquad \sum_{j=1}^{n} X_{fi} \; r_{ij} = X'_{i}, \text{ then}$$

$$(9) \qquad a_{ij} \; X'_{i} = T'$$

Equation (8) shows that we multiply each column of $(I - A)_T^{-1}$ by the new final demand associated with the corresponding row. Each column is then summed to obtain the new total gross output (X'_i).[1/] Finally, in equation (9), each column of the table of direct input coefficients is multiplied by the new total gross output (X'_i) for the corresponding row. The result is the new transactions Table T' which can be described by the following new balance equation:

$$(10) \qquad X'_i = \sum_{i=1}^{n} a_{ij} \; (X'_j) + X'_{f}, \quad (i = 1 \; ... \; n)$$

When the "dynamic" model is used in making long-range projections, the fixed technical coefficients - the a_{ij} of the original A matrix - are replaced by new coefficients computed from a sample of "best practice"

1/ To simplify the exposition we ignore certain inventory adjustments here which have to be made in practice.

establishments in each sector. All of the computational procedures described above remain unchanged, however. This could be symbolized by substituting a'_{ij} for a_{ij} in (10) indicating that all components of the balance equation are changed in the "dynamic" model.

6. DATA REQUIREMENTS

In this section a brief examination will be made of the sources and nature of the basic data to be obtained to provide the transactions table for the input-output model. This examination comprises two things. First, it comprises a table of estimated transactions for the Yukon that may be used to illustrate roughly the general nature of such a table for that economy. Second, it reviews broadly the nature and significance of the task of compiling an effective transactions table for such an input-output model.

Illustrative Yukon Transactions Table

In Table 5 is presented, on the basis of arbitrary estimates, an illustrative transactions table for the Yukon economy for the year 1966.[1/] This latest census year was selected because it provides more data on the Yukon economy than has generally been available in the past. Estimates of the total outputs of the industry sectors have been based generally on data provided in the statistical study of the Yukon economy.[2/] Distribution of inputs comprising

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1/ See Table 5, Appendix B.

2/ See Analysis of Statistics and Statistical Needs of the Yukon Territory, by Joan Gherson, a background study prepared as part of the Yukon Economic Studies, Ottawa, July, 1968.

these total outputs have been estimated where possible from Yukon data and where these were not available from percentages adapted from the pertinent data in the Dominion Bureau of Statistics transactions table for Canada.[1/] Imports have been estimated as a residual input. Data for final demand have been estimated partly from Yukon data, partly from knowledge of the industry and partly from data in the Canada input-output table.

Sources and Compilation

The estimates in Table 5 can be used only as illustrative. They are not suitable for an operative model. The data for a suitable operative model (whether input-output or aggregative) are not readily available as might seem to be implied by the terms of reference (see Preface). Moreover, the task of compiling precise and adequate data for an effective Yukon model should not be minimized. A substantial input of highly competent research will be required over a year or more, depending on the personnel devoted to the task.[2/] We would estimate that two to three man-years of competent senior economic research would do the job. A comprehensive study of each

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1/ Inter-Industry Flow of Goods and Services, Canada, 1949, Table 1, Dominion Bureau of Statistics, Cat. No. 13-513.

2/ There are several capable people who have already had considerable experience in directing these data compilations in Canada.

industrial sector would be required, as well as an analysis of inventory adjustments, imports, taxes, depreciation and wages and salaries. On the final demand side the analyses would include exports, personal consumption, government expenditures, and private capital formation. Each of these would require a special study. Since imports are so large a proportion of inputs into the industrial sectors in the Yukon a special effort to provide the data on the distribution of imports should be made. Several other industries in addition to those shown in Table 5 may eventually be considered for inclusion as separate sectors, especially those that might be expected to expand in the future.

The basic task still to be completed for the model is the preparation of a more adequately precise transactions table for the Yukon economy. This should not be nearly as great a task in the case of an economy with as few sectors as the Yukon as it is with the more diverse economies of most provinces. The sources and analytical approaches to be used in this compilation of transactions data have become fairly well established for the most part in the input-output models already established in Canada.[1/] Particular attention will need to be

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1/ Only a part of the description of sources and techniques for the data compilation has been published on Canadian input-output models. Yet it is available from unpublished sources and along with it there is a general readiness by those experienced in the field to co-operate and stimulate the work on new models.

given to those sectors which may need to be strengthened in the Yukon economy.

Finally, it may be desirable to note that the costs of completing the development of an operative input-output model for the Yukon are not large relatively. These costs are not likely to be as large as the cost of developing the time series and equations required for an aggregative model. And the input-output model will provide a great deal more of the economic information needed to guide public expenditure policies for the Yukon. Moreover, the costs of completing the input-output model are not large relative to the greater effectiveness and savings in the public expenditures that may be expected because of (a) the special insights into the economy provided by this model, (b) the greater preciseness with which interventions may be made, and (c) the greater understanding and confidence that is stimulated by the improved knowledge of the structure and workings of the economy that will stem from the development and operation of this model. At this stage in the development of the Yukon economy the contribution of an input-output model can be of very great benefit. It would provide an excellent tool in the development planning that seems necessary for the Yukon. For this reason it is particularly recommended at this time.

APPENDIX A

APPENDIX A

DEPARTMENT OF FINANCE, ANNUAL MODEL XIV, SEPTEMBER, 1966

$$(1)\quad CD(t) = \underset{(5.50)}{-.267} + \underset{(10.5)}{.130}\ YW^{na}(t) + \underset{(1.36)}{270}\ ID(t) + \underset{(.70)}{.135}\ ID(t-1) + \underset{(1.91)}{.159}\left[\rho(t-1) - \rho(t-2)\right] \underset{(4.16)}{-.287}\ d/p + u1$$

$$\bar{R}^2 = .982 \qquad Q = 0.97$$

$$(2)\quad CND(t) = \underset{(7.76)}{1.351} + \underset{(9.73)}{.345}\ YW^{na}(t) + \underset{(4.81)}{.349}\ CND(t-1) - \underset{(4.36)}{1.163}\ B(t-1)/L(t-1) + \underset{(1.25)}{.009}\ t + u2$$

$$\bar{R}^2 = .999 \qquad Q = 1.59$$

$$(3)\quad CS(t) = \underset{(2.50)}{.148} + \underset{(4.71)}{.219}\ YW^{na}(t) + \underset{(6.35)}{.580}\ CS(t-1) - \underset{(2.00)}{.014}\ t + u3$$

$$\bar{R}^2 = .999 \qquad Q = 1.33$$

$$(4)\quad \Delta H19(t) = \underset{(4.02)}{1.339} + \underset{(3.97)}{.137}\left[X(t) - X(t-1)\right] - \underset{(3.82)}{3.087}\left\{H19(t-1) / \left[CD(t-1) + CND(t-1) + G(t-1) + E(t-1)\right]\right\}$$

$$- \underset{(2.26)}{.006}\ t + \underset{(2.92)}{.596}\left[CD(t-1) - CD(t-2)\right] + u4$$

$$\bar{R}^2 = .703 \qquad Q = 1.82$$

$$(5)\quad w(t)\cdot N(t)\cdot h(t) / P(t) = \underset{(.44)}{-.022} + \underset{(13.9)}{.397}\ X(t) - \underset{(3.30)}{.254}\ X(t-1) + \underset{(5.52)}{.713}\ w(t-1)\cdot N(t-1)\cdot h(t-1) / P(t-1) + u5$$

$$\bar{R}^2 = .999 \qquad Q = 1.84$$

$$(6)\quad F1(t) = \underset{(1.19)}{.627} + \underset{(2.76)}{.405}\ CD(t) + \underset{(4.69)}{.263}\ CND(t) + \underset{(3.28)}{.379}\ \Delta H19(t) + \underset{(10.2)}{.988}\ IM(t) - \underset{(1.53)}{.784}\ P1/P - \underset{(3.18)}{.029}\ t + u6$$

$$\bar{R}^2 = .995 \qquad Q = 1.42$$

$$(7)\quad w(t) = \underset{(1.64)}{-.048} + \underset{(17.9)}{.910}\ w(t-1) + \underset{(.96)}{.032}\ N(t-1) + \underset{(2.55)}{.004}\ t + \underset{(4.05)}{.035}\ (1.0 - 100\ nU/nL)^{+} + u7$$

(8) $$h(t) = \underset{(.81)}{.069} + \underset{(25.4)}{.959}\, h(t-1) - \underset{(5.14)}{1.368} \left\{ \left[nL(t) + nY(t) - NA(t) - NG(t) - NE^{na}(t) - N(t) \right] / nL(t) \right\}$$
$$+ \underset{(6.07)}{1.587} \left[nU(t-1) / nL(t-1) \right] + u11$$

$\bar{R}^2 = .954 \qquad Q = 1.62$

(9) $$P(t) = \underset{(2.87)}{.084} + \underset{(8.96)}{.605}\, w(t) - \underset{(7.77)}{.528}\, w(t-1) + \underset{(9.70)}{.803}\, P(t-1) - \underset{(1.08)}{.071} \left\{ H19(t-1) \;/\; \left[CD(t-1) + CND(t-1) \right. \right.$$
$$\left. \left. + G(t-1) + E(t-1) \right] \right\} + u8$$

$\bar{R}^2 = .998 \qquad Q = 1.82$

(10) $$IPM(t) = \underset{(5.26)}{-1.176} + \underset{(7.72)}{.556}\, IPM(t-1) + \underset{(8.22)}{.521}\, X418(t-1) - \underset{(3.21)}{.254}\, X418(t-2) + \underset{(2.48)}{.317} \left[\rho(t-1) - \rho(t-2) \right] + \underset{(4.03)}{.581}\, UI12(t-1) + u9$$

$\bar{R}^2 = .991 \qquad Q = 1.82$

(11) $$IM(t) = \underset{(1.86)}{+.083} + \underset{(38.4)}{.498}\, IPM(t) + u10$$

$\bar{R}^2 = .978 \qquad Q = .41$

(12) $$YW^{na}(t) = w(t) \cdot N(t) \cdot h(t) / P(t) + RW(t) - TW(t) + WG(t) + WM(t)$$

(13) $$X(t) = CD(t) + CND(t) + CS(t) + ID(t) + IP(t) + IM(t) + \Delta H19(t) + \Delta H3(t) + E(t) - F1(t) - F^{s}(t)$$
$$- X^{a}(t) - G1 - E4(t) + F4(t) + R2(t) + G(t)$$

(14) $$\$J(t) = .500 \left[P(t) - P(t-1) \right] \cdot \Delta H19(t) + \left[P(t) - P(t-1) \right] \cdot H19(t-1)$$

(15) $$\Pi(t) = X^{a}(t) + G1(t) + E4(t) - F4(t) - D(t) - Z17(t) - R1(t) - w(t) \cdot N(t) \cdot h(t) / P(t) + X(t)$$
$$+ \$J(t) / P(t) - Ti(t) + Ts(t)$$

[illegible] $= -.[illegible] + .[illegible] \left[[illegible] - \$A(t) - \$V2(t) + \$dp(t) + \$\Pi(t) \right] - 1.084\ \$\Pi G(t)$

(1[illegible]) [illegible] $= .017 + .105\ \$TBM(t)$

(1[illegible]) \$[illegible] $= -.009 + .095\ \$TBM(t)$

(19) $IP(t) = IPM(t) - IM(t)$

(20) $I(t) = ID(t) + IP(t) + IM(t)$

(21) $\$TBV(t) = .329 + .265 \left[\$\Pi(t) - \$A(t) + \$G12(t) + \$df(t) \right] + .735 \left[\$V2(t) - \$dp(t) \right] + \$dp(t)$

(22) $X418(t) = \left\{ \left[\$\Pi(t) + \$TM3(t) + \$RM(t) + \$RV(t) + \$RA(t) - \$TM(t) - \$TV(t) - \$TA(t) - \$RW11(t) \right] / PPM(t) \right\} + D(t)$

(23) $C[illegible](t) = CD(t) + CND(t)$

(24) $\$Ti2(t) = .085\ \$F1(t)$

(25) $\$Ti3(t) = .028\ \$CG(t)$

(26) $\$Ti4(t) = .050 \left[\$CG(t) + \$I(t) + \$EI(t) \right]$

(27) $\$Ti6(t) = \$Ti1(t) + \$Ti5(t) + \$Ti2(t) + \$Ti3(t) + \$Ti4(t)$

(28) $\$Ti9(t) = .033\ \$CG(t)$

(29) $\$Ti14(t) = .040\ \$CG(t)$

(30) $$\$Ti16(t) = \sum_{k=7}^{15} \$Tik(t)$$

(31) $$\$Ti22(t) = \sum_{k=17}^{21} \$Tik(t)$$

(32) $\$Ti(t) = \$Ti6(t) + \$Ti16(t) + \$Ti22(t)$

(33) $\$TW1a^1(t) = .013 \left[w(t) \cdot N(t) \cdot h(t) + \$WG(t) + \$WM(t) \right]$

(34) $\$TW1a^2(t) = .0065 \left[w(t) \cdot N(t) \cdot h(t) + \$WG(t) + \$WM(t) \right]$

$$(36)\ \$TW1b^2(t) = .180\ \$WG^2(t)$$

$$(37)\ \$TW1b^3(t) = .022\ \$WG^3(t)$$

$$(38)\ \$TW1(t) = \$TW1a^1(t) + \$TW1a^2(t) + \$TW1b^1(t) + \$TW1b^2(t) + \$TW1b^3(t)$$

$$(39)\ \$TW2(t) = -.575\ \$TW2(t-1) + .149 \left[w(t)\cdot N(t)\cdot h(t) + \$WG(t) + \$WM(t) \right]$$

$$(40)\ \$TW2^1(t) = .825\ \$TW2(t)$$

$$(41)\ \$TW2^2(t) = .175\ \$TW2(t)$$

$$(42)\ \$TW3^1(t) = .0001 \left[w(t)\cdot N(t)\cdot h(t) + \$WG(t) + \$WM(t) \right]$$

$$(43)\ \$TW3^2(t) = .008 \left[w(t)\cdot N(t)\cdot h(t) + \$WG(t) + \$WM(t) \right]$$

$$(44)\ \$TW3^3(t) = .001 \left[w(t)\cdot N(t)\cdot h(t) + \$WG(t) + \$WM(t) \right]$$

$$(45)\ \$TW3(t) = \sum_{k=1}^{3} \$TW3^k(t)$$

$$(46)\ \$TW(t) = \$TW1(t) + \$TW2(t) + \$TW3(t)$$

$$(47)\ \$RW5(t) = -.006 + .489 \left\{ \left[nL(t) + nY(t) - NA(t) - NG(t) - NE^{na}(t) \right] \cdot h(t-1) - N(t)\cdot h(t) \right\}$$

$$(48)\ \$RW8(t) = .137 \quad ; \quad 1966,\ .145 \quad ; \quad 1967,\ .153$$

$$(49)\ \$RW9(t) = -.182 + .027\ N(t)\cdot h(t)$$

$$(50)\ \$RW(t) = \sum_{i=1}^{11} \$RWi(t) + \$RV2(t) + \$RV3(t) + \$RV5(t)$$

$$(51)\ \$TM1^1(t) = .310\ \$TBM(t)$$

$$(52)\ \$TM2(t) = .150 \left[\$df(t) + \$if(t) + \$G11(t) \right]$$

$$(53)\ \$TM(t) = \$TM1^1(t) + \$TM1^2(t) + \$TM2(t) + \$TM3 + \$TM4(t) + \$RW11(t)$$

$$(54)\ \$RM^1(t) = .520 \left[\$G12^1(t) - .160\ \$G11(t) \right] - \$GA(t) + .160\ \$G11(t)$$

$$(55)\ \$RM^2(t) = .520 \left[\$G12^2(t) - .500\ \$G11(t) \right] + .500\ \$G11(t)$$

(56) $\$RM^{3}(t) = \text{[illegible]}20 \left[\$G12^{3}(t) - .340\ \$G11(t) \right] + .340\ \$G11(t)$

(57) $\$RM(t) = \sum_{k=1}^{3} \$RM^{k}(t)$

(58) $\$TV1(t) = \text{[illegible]}\ \$TBV(t)$

(59) $\$TV1^{1}(t) = .825\ \$TV1(t)$

(60) $\$TV1^{2}(t) = .175\ \$TV1(t)$

(61) $\$TV(t) = \$TV1(t) + \$TV2(t)$

(62) $\$RV1^{1}(t) = .480 \left[\$G12^{1}(t) - .160\ \$G11(t) \right] + \$GA(t)$

(63) $\$RV1^{2}(t) = .480 \left[\$G12^{2}(t) - .500\ \$G11(t) \right]$

(64) $\$RV1^{3}(t) = .480 \left[\$G12^{3}(t) - .340\ \$G11(t) \right]$

(65) $\$RV1(t) = \sum_{k=1}^{3} \$RV1^{k}(t)$

(66) $\$RV(t) = \$RV1(t) + \$RV4(t)$

(67) $p15(t) = \left[p5(t)\cdot ID(t) + p6(t)\cdot IP(t) + p7(t)\cdot IM(t) \right] / \left[ID(t) + IP(t) + IM(t) \right]$

(68) $p16(t) = \left[p6(t)\cdot IP(t) + p7(t)\cdot IM(t) \right] / \left[IP(t) + IM(t) \right]$

(69) $$P(t) = \Big\{ p1(t)\cdot CND(t) + p2(t)\cdot CS(t) + p3(t)\cdot CD(t) + p4(t)\cdot G(t) + p15(t)\cdot \left[I(t) - D(t) \right] + p8(t)\cdot \Delta H19(t)$$
$$+ p9(t)\cdot \Delta H3(t) + p10(t)\cdot E(t) - p11(t)\cdot F1(t) - p12(t)\cdot F^{s}(t) + P(t)\cdot R2(t) \Big\}$$
$$\div \Big\{ CND(t) + CS(t) + CD(t) + I(t) + \Delta H19(t) + \Delta H3(t) + E(t) - F1(t) - F^{s}(t) + R2(t) - D(t) + G(t) \Big\}$$

(70) $nL = .382\ m1 + .883\ m2 + .977\ m3 + .917\ m4 + .266\ m5 + .300\ n1 + .515\ n2 + .325\ n3 + .330\ n4 + .061\ n5$

APPENDIX A

DEPARTMENT OF FINANCE, ANNUAL MODEL XIV

SEPTEMBER, 1966

GLOSSARY OF SYMBOLS

A	-	net income received by farm operators from farm production
B	-	household debt (excludes residential mortgages)
CD	-	personal expenditure on consumer durable goods
CG	-	CD + CND
CND	-	personal expenditure on consumer non-durable goods
CS	-	personal expenditure on consumer services
D	-	capital consumption allowances and miscellaneous valuation adjustments
df	-	dividends, Canadian corporations to non-residents
Dp	-	dividends, Canadian corporations to residents
d/p	-	credit condition variable
E	-	exports of goods and services
M	-	imports of goods and services
G	=	government expenditure on goods and services
G1	=	output of government services
G11	=	government debt interest paid to non-residents
G12	=	total government debt interest
$G12^1$	=	" " " " - federal
$G12^2$	=	" " " " - provincial
$G12^3$	=	" " " " - municipal
GA	=	interest on government annutiies
GNE	=	gross national expenditure
h	=	hours worked per year in the private non-agricultural sector
H3	=	farm inventories and grain in Commercial channels
H19	=	non-farm business inventories
I	=	business gross fixed capital formation
ID	=	new residential construction
if	=	interest payments to non-residents by private sector

IM = private investment in new machinery and equipment

IP = private investment in new non-residential construction

IPM = IP + IM

IVA = inventory valuation adjustment (= -J)

J = inventory valuation adjustment with sign changed

L = household liquid assets

[illegible] = male civilian non-institutional population - 14 years and over

m1 = " " " " - 14 to 19 years

m2 = " " " " - 20 to 24 years

m3 = male civilian non-institutional population - 25 to 44 years

m4 = " " " " - 45 to 64 years

m5 = " " " " - 65 and over

[illegible] (NNIFC + [illegible] - IVA - W) -V - A

[illegible] = female civilian non-institutional population - 14 years and over

[illegible] " " " " - 14 to 19 years

[illegible] = " " " " - 20 to 24 years

[illegible] " " " - 25 to 44 years

[illegible] " " " - 45 to 64 years

[illegible] " " " - 65 and over

[illegible] = civilian labour force

[illegible] = number unemployed

nY = number employed in remote areas

[illegible] = number of paid workers employed in the private non-agricultural sector

[illegible] = total agricultural employment

[illegible] = number of employers, own accounts and unpaid family workers - non-agriculture

NG = number of government employees

NNIFC = net national income at factor cost

$\underline{p^1}$ = price deflator (1957=1.0) - <u>CND</u>

$\underline{p^2}$ = " " " - <u>CS</u>

$\underline{p^3}$ = " " " - <u>CD</u>

$\underline{p^4}$ = " " " - <u>G</u>

$\underline{p^5}$ = " " " - <u>ID</u>

$\underline{p^6}$ = " " " - <u>IP</u>

$\underline{p^7}$ = " " " - <u>IM</u>

$\underline{p^8}$ = " " " - <u>ΔH19</u>

$\underline{p^9}$ = " " " - <u>ΔH3</u>

$\underline{p^{10}}$ = " " " - <u>E</u>

$\underline{p^{11}}$ = " " " - <u>F1</u>

$\underline{p^{12}}$ = " " " - $\underline{F^S}$

$\underline{p^{13}}$ = " " " - <u>E1</u>

$\underline{p^{14}}$ = " " " - <u>P1</u>

$\underline{p^{15}}$ = " " " - <u>ID</u>

$\underline{p^{16}}$ = " " " - <u>IPM</u>

[illegible] = price index of net national expenditure

[illegible] = price index of merchandise imports after all border crossing costs

<u>IPM</u> = <u>p16</u> = price index of <u>IPM</u>

<u>R1</u> = residual error of estimate for <u>GNP</u>

<u>R2</u> = " " " " <u>GNE</u>

<u>RA</u> = total transfers to agricultural sector

<u>RM</u> = transfers to corporate sector = $\sum_{k=1}^{3} \underline{RM^k}$

$\underline{RM^1}$ = transfers to corporate sector - federal

$\underline{RM^2}$ = " " " " - provincial

$\underline{RM^3}$ = " " " " - municipal

RV = transfers to personal non-wage, non-farm sector

RV1 = government debt interest to resident persons

$RV1^1$ = " " " " " " - federal

$RV1^2$ = " " " " " " - provincial

$RV1^3$ = " " " " " " - municipal

RV2 = grants to universities

RV3 = assistance to immigrants

[illegible] = miscellaneous transfers to the personal non-wage, non-farm sector

RV5 = grants to private non-commercial institutions

[illegible] = $\sum_{i=i}^{11}$ RWi + RV2 + RV3 + RV5 = total transfers to wage sector

RW1 = family allowances

RW2 = re-establishment credit and rehabilitation benefits

[illegible] = pensions world wars I and II

[illegible] = war veterans' allowances

RW5 = unemployment insurance benefits

[illegible] pensions to government employees

[illegible] old age security fund payments

[illegible] = direct relief

[illegible] = workmens' compensation benefits

[illegible] = mothers' and dependents' allowances

[illegible] charitable contributions of corporations

t = time - number of years from 1926

[illegible] direct tax base for corporate sector

[illegible] = direct tax base for personal non-wage, non-farm sector

Ti = indirect taxes - total = Ti6 + Ti16 + Ti22

Ti1 = indirect taxes - federal: banks and insurance companies

Ti2 = " " " customs import duties

Ti3 = " " " excise duties

Ti4 = " " " excise taxes

Ti5 = " " " miscellaneous

Ti6 = total federal indirect taxes = $\sum_{K=1}^{5} \text{TiK}$

Ti7 = indirect taxes - provincial: amusement

Ti8 = " " " corporation tax (not on profits)

Ti9 = " " " gasoline

Ti10 = " " " licences, fees and permits

Ti11 = " " " motor vehicle licences and permits for business

Ti12 = " " " miscellaneous taxes on natural resources

Ti13 = " " " real and personal property

Ti14 = " " " retail sales tax (including liquor and tobacco)

Ti15 = " " " miscellaneous

Ti16 = total provincial indirect taxes = $\sum_{K=7}^{15} \text{TiK}$

Ti17 = indirect taxes - municipal: amusement

Ti18 = " " " licences, fees and permits

Ti19 = " " " real and personal property

Ti20 = " " " retail sales tax

Ti21 = " " " miscellaneous

Ti22 = total municipal indirect taxes = $\sum_{K=17}^{21} \text{TiK}$

TA = transfers from the agricultural sector

TC = " " " corporate sector

TD1 = corporate direct tax

TD11 = " " " - federal

TD12 = " " " - provincial

TD2 = withholding tax

TM3 = government trading profits

TM4 = government interest revenue

Ts = subsidies

TV = transfers from the personal non-wage non-farm sector

TV1 = non-wage portion of personal income tax

$TV1^1$ = " " " " " " - federal

$TV1^2$ = " " " " " " - provincial

TV2 = succession duties

TW = TW1 + TW2 + TW3 = transfers from wage sector

TW1 = employer and employee contributions to social insurance and government pension funds

TW1a = same as TW1 but excluding contributions to public service pensions

$TW1a^1$ = contributions to unemployment insurance

$TW1a^2$ = contributions to workmen's compensation and industrial employees' vacations

TW1b = contributions to public service pensions

$TW1b^1$ = " " " " " - federal

$TW1b^2$ = " " " " " - provincial

$TW1b^3$ = " " " " " - municipal

TW2 = wage portion of personal income tax

$TW2^1$ = " " " " " " - federal

$TW2^2$ = " " " " " " - provincial

TW3 = miscellaneous personal direct tax = $\sum_{k=1}^{3} TW3^k$

$TW3^1$ = miscellaneous personal direct tax - federal

$TW3^2$ = " " " " - provincial

$TW3^3$ = " " " " - municipal

[illegible]i = (defined for i-th equation) = non-systematic component of the variable on the left hand side of the equality sign, i.e. the dependent variable

UI12 = unfilled orders from private sector for the non-residential construction industry

V = personal non-farm non-wage income

V2 = interest, dividends and net rental income of persons

w = average hourly earnings of paid workers in private non-farm sector

W = W_p^{na} + WA + WG + WM

WA = agricultural wage bill

WG = government civilian wage bill

WG^1 = " " " " - federal

WG^2 = " " " " - provincial

WG^3 = " " " " - municipal

WM = military pay and allowances

W_p^{na} = wage-salary bill for the private non-agricultural sector

X = GDP - private non-agriculture

X^a = GDP - agriculture

X418 = \$(Π + TM3 + RM + RV - TM - TV - TA - RW11)/PPM + D

YM = disposable corporate income

YV = disposable personal non-farm non-wage income

YW = disposable wage income

YW^{na} = disposable non-farm wage income

Z17 = total government wage bill plus agricultural wage bill

Δ = one - period change

Π = X^a + G1 + E4 - F4 - D - Z17 - R1 - w.N.h/P + X + \$J/P - Ti + Ts

ΠG = TM3 + TM4

ρ = price index of industrial common stocks

\$ = denotes valuation in current dollars of variable which it precedes

$(4.0 - 100\ nU/nL)^+$ - the positive branch, only, of this variable, i.e. if 4.0 <100 nU/nL the variable is inactive (zero)

APPENDIX B

APPENDIX B

TABLE 5 - ILLUSTRATIVE YUKON TRANSACTIONS TABLE, 1966[1/]*

	Mining	Forestry	Other Primary	Manufacturing	Construction	Trptn., Stge. & Trade	Electric Power	Services	Final Output: Domestic	Final Output: Export	Total Output
(Producers' prices in thousands of dollars)											
Mining	50[2/]	-	-	-	20[2/]	-	-	30[2/]	-	10,000[3/]	10,100[4/]
Forestry	100[5/]	-	-	-	330[5/]	-	45[5/]	-	125[3/]	-	600[6/]
Other Primary	-	-	1[2/]	-	-	-	-	-	62[3/]	50[3/]	113[7/]
Manufacturing	-	-	-	55[2/]	100[2/]	45[2/]	-	100[2/]	300[3/]	-	600[8/]
Construction	105[2/]	10[2/]	5[2/]	10[2/]	20[3/]	1,250[2/]	180[2/]	1,720[3/]	15,900[2/]	-	19,200[9/]
Trptn., Stge. and Trade	800[2/]	50[2/]	-	-	4,600[2/]	4,700[2/]	-	1,400[2/]	6,700[3/]	7,250[3/]	25,500[10/]
Electric Power	690[3/]	20[3/]	-	70[3/]	70[3/]	90[3/]	370[3/]	230[3/]	760[3/]	-	2,300[11/]
Services	400[2/]	-	-	-	550[2/]	700[2/]	-	950[2/]	11,000[3/]	-	13,600[12/]
Imports	955[3/]	70[3/]	31[3/]	250[3/]	6,510[3/]	715[3/]	405[3/]	370[3/]			
Wages and salaries	4,100[13/]	200[14/]	10[15/]	150[14/]	5,200[15/]	10,500[15/]	500[16/]	6,300[15/]			
Other Primary Inputs	2,900[17/]	250[17/]	66[17/]	65[17/]	1,800[17/]	7,500[17/]	800[17/]	2,500[17/]			
Total Primary Inputs	7,000[18/]	450[15/]	76[15/]	215[15/]	7,000[15/]	18,000[15/]	1,300[15/]	8,800[15/]			
Total Input	10,100[4/]	600[6/]	113[7/]	600[8/]	19,200[9/]	25,500[10/]	2,300[11/]	13,600[12/]			

FOOTNOTES TO TABLE 5

1/ References on which this table are based may be found listed in the bibliography. In the following footnotes these references are denoted by bracketed numbers corresponding to their number as listed in the bibliography.

2/ Estimates based on input-output ratios for Canada (7) adjusted by data from the Yukon studies (4, 11) and supporting background data.

3/ Estimates based on data from the Yukon economic study (4).

4/ Estimated from 1966 value of mineral shipments adjusted to value of production on the basis of average 1961-64 adjustment (11, Tables 33 and 42).

5/ (11, Table 43)

6/ Adjusted from 1966-67 fiscal year to 1966 calendar year (11, Table 44).

7/ "Other primary" comprises trapping, fishing and agriculture (11, Tables 46, 49 and 53).

8/ Estimated average for 1961-66 (11, Table 69).

9/ (11, Table 55)

10/ Comprises estimated 1966 value of transportation (11, Table 72) and gross retail margin (26.9 per cent for Canada in 1961) on estimated Yukon retail sales (11, Table 70, Estimate II).

11/ Preliminary estimate, 1966, Dominion Bureau of Statistics.

12/ Comprises estimated 1966 receipts of the service trades, Yukon Territory (11, Table 71) and 1966 value of public services represented by the estimated operating expenditures of the Yukon Territorial Government, 1966 (11, Table 80).

13/ Average 1961-64 salaries and wages adjusted by 1966 estimate of value of production (11, Table 42).

14/ Average 1961-63 total payrolls based on Workmen's Compensation records adjusted by 1966 value of production.

15/ Estimates based on input-output ratios for Canada (7) adjusted according to the background research for the Yukon study (4).

16/ Estimate based on 1959-64 total wages and salaries adjusted by 1966 total value of sales to ultimate customers (11, Table 54).

17/ Difference between the 1966 wages and salaries estimate and the total primary inputs estimate.

18/ Average 1961-64 net value added adjusted by 1966 value of production data (11, Table 42).

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